Operational Research

# Midterm exam preparation

1. You are given a linear program:

max z = 3 x1 + 4 x2

2 x1 + 3 x2 ≤ 5

5 x1 - 2 x2 ≥ 3

x1 ≥ 0, -∞ < x2 < +∞

1. Solve the problem graphically
2. Solve the problem using simplex method
3. For a given linear program:

min Z = 3 x1 + 5 x2

with respect to: 2 x1 + x2 ≤ 10

6 x1 – 2 x2 ≥ 9

2 x1 + x2 = 4

3 x1 – 4 x2 ≥ -3

x1 ≥ 0

x2 ≥ 0

* 1. Formulate model in canonical form
  2. Formulate a dual model

1. Solve the dual of a given linear program.

max z = x1 – 3x2

x1 + x2 = 5

x1 – x2 ≤ 10

x1 ≥ -5, ∝ > x2 > -∝

1. Next table states transportation problem where A, B, C and D are sources and 1, 2, 3 and 4 destinations for some goods:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | Available quantities [t] |
| A | 10 | 30 | 70 | 10 | 30 |
| B | 30 | 70 | 30 | 20 | 40 |
| C | 20 | 50 | 40 | 5 | 80 |
| D | 50 | 10 | 25 | 80 | 50 |
| Requirements [t] | 20 | 50 | 60 | 50 |  |

Solve the given problem using MODI method.

1. Some chemical mixture components specifications are:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Raw material [kg] | Useful ingredients [% weight] | | | | | Price [kn/kg] |
| A | B | C | D | E |
| S1 | 3 | 5 | 20 | 17 | 35 | 10 |
| S2 | 16 | 23 | 12 | 5 | 4 | 15 |

You are to find the cheapest mixture which satisfies constraints on percentage composition:

* It has to containt at least 10% of ingredients A and B, combined
* There has to be at least twice as much of C than D
* It has to containt no more of D and E combined than A,B and C combined
* The mixture has to contain at least 50% of useful ingredients

Set (but don't solve)the linear programming model.

1. Some student listens to 3 courses. Table states the maximum points available through specific activities. For every student we assume that each invested hour corresponds to one achieved point. Course pass minimum is 50 points.

|  |  |  |  |
| --- | --- | --- | --- |
| **Type of activity *i*/ Course *j*** | **1** | **2** | **3** |
| **1. Short exams** | 10 | 8 | 12 |
| **2. Homeworks** | 20 | 15 | 12 |
| **3. Midterms** | 35 | 42 | 30 |
| **4. Final** | 25 | 35 | 40 |
| **5. Oral** | 10 | 0 | 6 |
| **Sum** |  |  |  |

* 1. Set linear programming model which would minimize student's invested time needed to pass all three courses. Instruction: X11 [h] should mark hours invested in course 1 short exams , X12 hours invested in course 2 short exams, …, X53 hours invested in preparation for course 3 oral exam
  2. Set linear programming model for the case that student wants to invest no more than 200 working hours but wants to minimize the risk of not passing or getting a lower grade on one of the courses

1. Formulate (but do not solve!) a linear program for the next manufacturing model:

Input raw material can be processed on 2 distinct machines which results in 4 halfproducts which are losslessly finalized on machine #3into 4 final products. Each machine works only with one material at a time.

2 [kg/h]

1 [kg/h]

0,5 [kg/h]

4 [kg/h]

0,25 [kg/h]

0,1 [kg/h]

0,2 [kg/h]

1 [kg/h]

X1[kg]; 6[HRK/kg]

X2[kg]; 8[HRK/kg]

X3[kg]; 10[HRK/kg]

X4[kg]; 4[HRK/kg]

≤ 10 [h]

≤ 20 [h]

≤ 5 [h]

Raw material costs are 50% of the final product selling price and machine operational costs are negligible.

Mass of every of final products should be no less than 10% of the total final products mass.